Trade Costs and Endogenous Nontradability in a Model with Sectoral and Firm-Level Heterogeneity

Manoj Atolia*
Florida State University

May 11, 2017

Abstract

The paper takes a first step in the direction of simultaneously incorporating sectoral and firm-level heterogeneity in the models of international trade and macroeconomics in a tractable manner: without increasing the complexity of numerical computations compared to the existing models with heterogeneity in one dimension. In a model with sectoral heterogeneity in trade costs and firm-level heterogeneity in productivity, introducing one source of heterogeneity at a time and piecing together the results implies that, on reduction in trade costs, more goods and more varieties of every tradable good become traded. However, in the correctly specified model with simultaneous heterogeneity in both dimensions, although more goods do indeed become tradable, but for more than 50% of the previously traded goods, the number of traded varieties *falls*. The model also reconciles apparently contrasting predictions for the differences in the deviation of domestic price from the world price for the traded and nontraded goods when heterogeneity is introduced, one dimension at a time.

Keywords: Heterogeneity, curse of dimensionality, endogenous nontradability, endogenous tradability, trade costs, firm-level productivity differences
JEL Classification: F11; F12; F41

1 Introduction

This paper seeks to add a new insight to the simple and clever ways that have been suggested and currently being used in the literature to incorporate sectoral and firm-level heterogene-

*Department of Economics, Florida State University, Tallahassee, FL 32306, U.S.A. Telephone: 850-644-7088. Email: matolia@fsu.edu.

ity to analyze issues in international trade and macroeconomics. Eaton and Kortum (2002) and Melitz (2003) incorporate firm-level heterogeneity in productivity to analyze trade flows. Similar efforts have been made to find answers to some of the basic questions facing open economy macroeconomics. Bergin and Glick (2007a, 2007b, 2009) introduce sectoral heterogeneity in trade costs and productivity to analyze macroeconomic implications of endogenous nontradedness. In addition, Ghironi and Melitz (2005) have used the setup in Melitz (2003) to address the same issue.

Following the seminal contribution of Dornbusch, Fischer, and Samuelson (1977), all these papers have bought analytical tractability by utilizing the 'power of the continuum' to simplify analysis. In particular, these papers incorporate heterogeneity in one dimension. However, for many issues of interest it would be of importance to incorporate heterogeneity both across sectors and across firms within sectors. For example, consider the issue of endogenous nontradability. The focus in Bergin and Glick is on cross-sectoral variations in tradability, whereas in Ghironi and Melitz (2005) it is on the within-sector determination of "tradedness" based on firm-level variations in productivity. Both strands of literature recognize the complementary nature of their approach but no attempt has so far been made to incorporate both elements of nontradedness in a single tractable model.

The paper takes a first step in the direction of incorporating heterogeneity in both dimensions in a tractable manner that is fairly general and amenable to use in many other situations. The tractability of the approach relies on the key insight of this paper which avoids the curse of dimensionality and the increase in complexity of numerical computation of the equilibrium, compared to existing models with heterogeneity in one dimension. This insight, which allows such gain in tractability, again exploits the power of the continuum. Recall, Dornbusch, Fischer, and Samuelson (1977) sidestepped the knotty problem of the marginal good separating the nontradables and tradables (or importables and exportables) by going from a large number of goods to a continuum of goods, where the marginal good can be nontraded or traded without affecting the equilibrium. Further, by assuming relative productivity to be a continuous function of the relevant goods index, it simultaneously provided an exact link (via an equality) between prices of the traded and nontraded goods.

To see how this idea extends naturally to two dimensions, consider a small economy model in which productivity varies across firms (varieties) and trade costs vary across sectors (goods) in a continuous manner with the respective indices. This is a generalization of the set up in Bergin and Glick (2009), who only consider sectoral differences in trade costs (based on Hummels, 1999 and 2001), to include firm-level productivity differences analyzed by Melitz (2003). This is also an empirically relevant case to consider following the empirical evidence on substantial productivity differences within narrowly defined industries.

In this set up, given equilibrium prices, for each sector there is a marginal nontraded variety and all varieties with productivity higher than this marginal variety are exported. Further, as trade costs vary continuously with sectoral indices, so does this marginal traded variety. More importantly, it is possible to derive an *analytical* expression for this relationship between a sector's index and the index of its marginally nontraded variety which we call the *marginal variety frontier*. This analytical characterization of marginal variety frontier (see equation (27)) considerably simplifies the computation of the equilibrium. For example, to solve for the steady state, all one needs to do is to solve one nonlinear equation in one unknown (see equation (41)), just as in the model without firm-level heterogeneity in Bergin and Glick (2009).[1]

In Bergin and Glick, to solve for the steady state, the computation algorithm would make a guess for the index of the marginally nontraded good. In our model with a continuum of sectors/goods and a continuum of varieties in each sector, at first it appears that computational algorithm would need to make a guess for the indices of marginally nontraded variety of each good. However, in light of the analytical characterization of the marginal variety frontier, this turns out to be unnecessary: the computational algorithm needs to guess index for the marginal nontraded variety for just one sector. Thereafter, the marginal variety frontier allows solving for marginal nontraded varieties for all other sectors analytically, resulting in the avoidance of curse of dimensionality.

While the paper shows that it is possible to easily incorporate heterogeneity in two dimensions, is it really necessary? It turns out that in the most natural and empirically relevant generalization of Bergin and Glick set up described above, the failure to simultaneously incorporate heterogeneity in two dimensions results in faulty analysis. The object of analysis is the response of (endogenous) nontradedness of various goods and varieties of goods to a change in trade costs. If one considers heterogeneity in one dimension at a time and pieces together the results, one would be erroneously conclude that a reduction in trade costs makes more goods and more varieties of every tradable good traded. However, in the correctly specified model with heterogeneity in both dimensions, one finds that whereas more goods indeed become tradable but not every traded sector experiences an increase in the number of varieties that are traded. In fact, in almost 50% of the previously traded sectors the number of traded varieties *falls*. The result is shown to be quite robust to a range of plausible parameter values and introduction of production in the economy. An example with similar flavor is provided by Chaney (2008) which examines the sensitivity of

[1] On the contrary, if we considered a model with s sectors and f firms in each sector, equilibrium computation would require solving for sf unknowns. Even, if one allowed a continuum of firms (sectors), one would need to solve for s (f) unknowns.

the impact of trade barriers on trade flows to the elasticity of substitution among goods (see also Baldwin and Forslid, 2010). It finds the elasticity of substitution has opposing effects along the intensive and extensive margins. Like this paper, Chaney (2008) has a continuum of firms in each sector with Pareto distribution of productivity, but unlike this paper has a finite number of sectors.

The model of this paper also, in a natural way, reconciles the contrasting predictions of models with heterogeneity only in trade costs or productivity about the differences in the deviation of domestic price from the world price for the traded and nontraded goods. At the economy level, these deviations are shown to be dictated by heterogeneity in trade costs consistent with evidence in Crucini, Telmer, and Zachariadis (2005): the deviations from the world price are larger for nontraded goods. The heterogeneity in productivity a la Dornbusch *et al.* (1977) is relevant as well, but it is shown to operate at sector level and implies that, for a given good, deviation of domestic price from the world price is smaller for nontraded varieties than for the traded varieties.

While the paper deals with specific types of sectoral and firm-level differences, namely trade costs and productivity, there are other relevant combinations of heterogeneity across sectors and firms that can be analyzed using the same idea. For example, following Bernard, Redding, and Schott (2007), one can have sectoral heterogeneity in factor intensity coupled with firm-level differences in productivity. Further, the method extends in a straightforward manner to a two country setting.

There is a burgeoning literature studying impact of trade on business cycle comovement, both in two/multi-country and small-open economy settings. For example, Johnson (2014), in a model with finite number of sectors, studies the role of trade in intermediate inputs to replicate quantitative magnitude of empirical correlation between bilateral trade and GDP comovement, the trade-comovement puzzle. The reduction of computational complexity makes the model of this paper a viable choice to study such issues along with its sectoral and firm-level implications.

In the remaining portion of the paper, section 2 lays out and solves the small open economy model with endowment. The analytical details pertaining to solving for the equilibrium and how power of continuum may be used to simplify computations are outlined in section 3. Section 4 numerically solves for the equilibrium of the small-open economy model with endowment. In section 5, results are presented for sectoral variations in the nontradability in response to a reduction in trade costs. Section 6 briefly considers the two country model. Section 7 concludes. The Appendix A provides the details for solving a small open economy model with production whereas the details for the two country model are contained in Appendix B.

2 A Small Open Economy with Endowment

To illustrate the key results in the simplest setting, we consider a small-open, endowment economy that has a continuum of sectors or goods indexed by $i \in [0,1]$. In each sector i, there is a continuum of firms indexed by $j \in [0,1]$ that produce a particular variety of good i. Thus, j may refer to the firm or the variety the firm produces. For ease of exposition, we may, occasionally, also call variety j of good i as good (i,j). The economy has fixed endowment of goods. All the goods can be potentially exported with their world prices given by $p^*_{i,j}$. Besides consuming home goods $(i,j) \in [0,1]^2$, the economy also imports, at price p_F, a composite foreign good F; its quantity being consumed is denoted by c_F.[2] Let $y_{i,j}$, $c_{i,j}$, and $p_{i,j}$ represent the endowment, consumption, and domestic price of home good (i,j).

The firm-level heterogeneity in productivity across varieties of a good i is introduced by normalizing world price of all its varieties, $p^*_{i,j}$ to $p^* \equiv 1$ and having the following distribution of endowment of the varieties:

$$y_{i,j} = y\ j^{\beta_a}, \quad y > 0,\ \beta_a \geq 0. \tag{1}$$

Thus, the economy has a higher endowment of the variety with higher index j. This is similar to having a higher productivity for the variety with higher index j in a production economy. The model with production is described in Appendix A. As $y_{i,j}$ is independent of i, the endowment of varieties is symmetric across industries or sectors and rises with elasticity β_a with j.

However, different industries face differing 'iceberg' trade costs given by the following distribution as in Bergin and Glick (2009):

$$1 + \tau_i = \alpha i^{-\beta_c}, \quad \alpha > 1,\ \beta_c \geq 0, \tag{2}$$

so that only one unit of good i reaches its destination when $1 + \tau_i$ units are exported. Thus, all varieties of a particular good i face same trade costs and the export price (the price received by Home exporters) of good (i,j) is independent of j and is given by

$$p_{i,j} = \frac{p^*}{1+\tau_i} = \frac{p^*}{\alpha} i^{\beta_c}. \tag{3}$$

From (2) it is also clear that trade costs fall with i with elasticity β_c. In particular, goods with

[2] Since world price of imported goods does not change, such aggregation of imported goods is appropriate in light of Hicks' Substitution Theorem.

higher i are more tradable. Since world prices of all home goods is identical, (2) also implies that goods with lower i have lower domestic price to offset higher trade costs. Furthermore, a higher β_c implies that export price (or equivalently tradability) of a good falls rapidly as i falls below 1.

The aggregate consumption (c) is given by

$$c = \frac{c_H^{\theta} c_F^{1-\theta}}{\theta^{\theta} (1-\theta)^{1-\theta}}, \tag{4}$$

where c_H is the index of consumption of home goods (i, j) that has consumption share of θ. The c_H, in turn, is defined by

$$c_H^{1-\frac{1}{\gamma}} = \int_0^1 c_i^{1-\frac{1}{\gamma}} di, \tag{5}$$

where $\gamma > 1$ is the elasticity of substitution across goods produced by different sectors and c_i is the index of consumption of good i given by

$$\begin{aligned} c_i^{1-\frac{1}{\phi}} &= \int_0^{n_i} c_{i,j}^{1-\frac{1}{\phi}} dj + \int_{n_i}^1 c_{i,j}^{1-\frac{1}{\phi}} dj, \\ &= n_i \left(\frac{c_{i,N}}{n_i}\right)^{1-\frac{1}{\phi}} + (1-n_i)\left(\frac{c_{i,T}}{1-n_i}\right)^{1-\frac{1}{\phi}}, \end{aligned} \tag{6}$$

where $\phi > \gamma > 1$ is the elasticity of substitution among varieties of a good and

$$c_{i,N} = n_i \left[\frac{1}{n_i} \int_0^{n_i} c_{i,j}^{1-\frac{1}{\phi}} dj\right]^{\frac{1}{1-\frac{1}{\phi}}}, \tag{7}$$

$$c_{i,T} = (1-n_i) \left[\frac{1}{1-n_i} \int_{n_i}^1 c_{i,j}^{1-\frac{1}{\phi}} dj\right]^{\frac{1}{1-\frac{1}{\phi}}}. \tag{8}$$

The above scheme of aggregation asserts that in every sector, i, there is a marginally non-traded variety $n_i \in [0, 1]$ so that varieties $j \in [0, n_i]$ are nontraded and varieties $j \in (n_i, 1]$ are traded. This assertion is proved subsequently. It may be noted that $c_{i,N}$ and $c_{i,T}$ have the interpretation of 'aggregate' consumption of traded and nontraded varieties of good i and, therefore, $c_{i,N}/n_i$ and $c_{i,N}/(1-n_i)$ have the interpretation of 'average' consumption of traded and nontraded varieties.

The consumption-based price indices for the above defined consumption aggregates follow immediately

$$
\begin{aligned}
p &= p_H^{\theta} p_F^{1-\theta}, && (9)\\
p_H^{1-\gamma} &= \int_0^1 p_i^{1-\gamma} di, && (10)\\
p_i^{1-\phi} &= \int_0^{n_i} p_{i,j}^{1-\phi} dj + \int_{n_i}^1 p_{i,j}^{1-\phi} dj,\\
&= n_i p_{i,N}^{1-\phi} + (1-n_i)\, p_{i,T}^{1-\phi}. && (11)
\end{aligned}
$$

Here p is the aggregate price level, p_H is the price index of home goods, and p_i is the price index of home good i. The price indices of nontraded and traded varieties of good i, $p_{i,N}$ and $p_{i,T}$ are

$$
\begin{aligned}
p_{i,N} &= \left[\frac{1}{n_i}\int_0^{n_i} p_{i,j}^{1-\phi} dj\right]^{\frac{1}{1-\phi}}, && (12)\\
p_{i,T} &= \left[\frac{1}{1-n_i}\int_{n_i}^{1} p_{i,j}^{1-\phi} dj\right]^{\frac{1}{1-\phi}}. && (13)
\end{aligned}
$$

3 Solving the Model

This section outlines the procedure to solve for the equilibrium of the endowment economy of Section 2. As in a model with heterogeneity in one dimension in Bergin and Glick (2009), the procedure describe here generalizes in a straightforward manner to an economy with production (see Appendix A).

3.1 Solving for the Prices and Consumption of Traded Goods

To solve for the equilibrium, we begin by proving the existence of a marginal nontraded variety for an exported good that formed the basis of the aggregation scheme in (6-8).

3.1.1 The Marginal Nontraded Variety

Consider the decision to export good variety j of good i. This variety of good i will be not be exported if the price it will fetch, in domestic market in the absence of export, is more than its export price.[3] It may be mentioned that with a continuum of varieties, the decision to export or not export variety j does not affect its price. For two varieties j and k of good i that are not exported, their prices are related via consumer optimization to their

[3]If a variety is exported, its domestic price equals the export price. See following discussion.

endowment. In particular, optimal consumption choice implies

$$\frac{p_{i,j}}{p_{i,k}} = \left[\frac{y_{i,j}}{y_{i,k}}\right]^{-\frac{1}{\phi}} = \left[\frac{yj^{\beta_a}}{yk^{\beta_a}}\right]^{-\frac{1}{\phi}} = \left[\frac{j}{k}\right]^{-\frac{\beta_a}{\phi}}. \tag{14}$$

From this it follows easily that if $k < j$ and variety j is nontraded then so is variety k.[4] Thus, there exist a *marginal nontraded variety* $n_i \in (0, 1]$ such that varieties $j \in [0, n_i]$ are nontraded and varieties $j \in (n_i, 1]$ are traded and n_i is also the share of nontraded varieties for good i.[5] The assumptions of a continuum of varieties of a good and the continuity of endowment of varieties over this continuum play a crucial in establishing the existence of n_i.

3.1.2 Prices

The fact that, with a continuum of varieties, a variety may be exported or consumed entirely domestically without affecting the equilibrium, when applied to marginally traded variety, implies that its domestic price must equal the export price

$$p_{n_i} = \frac{p^*}{\alpha} i^{\beta_c}. \tag{15}$$

This *marginal nontraded variety condition* provides a crucial link between the prices of traded and nontraded varieties of good i. It may also be mentioned that export price is same for all (exported) varieties of a good as trade costs are same for all varieties.

The existence of the marginal nontraded good (or variety) for which (15) holds as equality is key to the analytical simplification achieved by Dornbusch, Fischer, and Samuelson (1977). It avoids the need to consider conditions based on inequalities to establish the boundary between the traded and the nontraded varieties. As mentioned earlier, this result hinges critically on the assumptions of a continuum of varieties of a good and the continuity of endowment of varieties over this continuum. These assumptions ensure that domestic price of varieties a good i is a continuous function of j.

Let $B \subset [0, 1]$ be the set of industries for which the marginal nontraded variety condition holds for some $n_i \in (0, 1]$. Then, for $i \in B$, the condition in (15) pins down $p_{i,N}$, as from

[4]To see this, first recall that export price of all varieties of good i is $p^* i^{\beta_c}/\alpha$. Next, from (14), we have that $p_{i,j} < p_{i,k}$. The fact that variety j is nontraded implies $p_{i,j} > p^* i^{\beta_c}/\alpha$, and therefore, $p_{i,k} > p^* i^{\beta_c}/\alpha$. Hence, variety k is nontraded as well.

[5]If any variety $j > 0$ has positive price (which is guaranteed by the consumption optimization) then variety 0 has infinite domestic price and hence is never exported. Thus, $n_i > 0$.

(12) we have

$$
\begin{aligned}
p_{i,N} &= \left[\frac{1}{n_i}\int_0^{n_i} p_{i,j}^{1-\phi} dj\right]^{\frac{1}{1-\phi}} = \left[\frac{1}{n_i}\int_0^{n_i} \left(\frac{j}{n_i}\right)^{-\frac{\beta_a(1-\phi)}{\phi}} p_{n_i}^{1-\phi} dj\right]^{\frac{1}{1-\phi}} \\
&= \frac{p^* i^{\beta_c}}{\alpha}\left[1+\beta_a\frac{\phi-1}{\phi}\right]^{\frac{1}{\phi-1}}. \qquad (16)
\end{aligned}
$$

Similarly, the price index of traded varieties of good $i \in B$, $p_{i,T}$ in (13) simplifies to

$$
p_{i,T} = \frac{p^*}{\alpha} i^{\beta_c}. \qquad (17)
$$

Thus, from (16) and (17), the relative price of nontraded varieties equals

$$
\frac{p_{i,N}}{p_{i,T}} = \left[1+\beta_a\frac{\phi-1}{\phi}\right]^{\frac{1}{\phi-1}}, \qquad (18)
$$

and is independent of $i \in B$. Further, from (11), we get

$$
\left[\frac{p_i}{p_{i,T}}\right]^{1-\phi} = n_i\left[\frac{p_{i,N}}{p_{i,T}}\right]^{1-\phi} + 1 - n_i = \frac{n_i}{1+\beta_a\frac{\phi-1}{\phi}} + 1 - n_i \equiv 1 - wn_i,
$$

which using (17) gives

$$
p_i = \frac{p^*}{\alpha}\frac{i^{\beta_c}}{[1-wn_i]^{\frac{1}{\phi-1}}}, \quad i \in B \qquad (19)
$$

where

$$
w = \frac{\beta_a\frac{\phi-1}{\phi}}{1+\beta_a\frac{\phi-1}{\phi}} < 1. \qquad (20)
$$

3.1.3 Consumption

To solve for consumption, first use (7) to obtain

$$
\begin{aligned}
c_{i,N} &= n_i\left[\frac{1}{n_i}\int_0^{n_i} c_{i,j}^{1-\frac{1}{\phi}} dj\right]^{\frac{1}{1-\frac{1}{\phi}}} = n_i\left[\frac{1}{n_i}\int_0^{n_i} (yj^{\beta_a})^{1-\frac{1}{\phi}} dj\right]^{\frac{1}{1-\frac{1}{\phi}}} \\
&= \frac{yn_i^{1+\beta_a}}{\left[1+\beta_a\frac{\phi-1}{\phi}\right]^{\frac{1}{1-\frac{1}{\phi}}}}. \qquad (21)
\end{aligned}
$$

For determining $c_{i,T}$, consider optimal allocation of consumption between a traded variety j, and the marginally nontraded variety n_i of good i, which implies

$$\frac{c_{i,j}}{c_{i,n_i}} = \left[\frac{p_{i,j}}{p_{i,n_i}}\right]^{-\phi} = 1, \quad j \in (n_i, 1], \tag{22}$$

where last equality follows from the fact that prices of all traded varieties are same as that of the marginal nontraded variety. As a result, we have[6]

$$c_{i,j} = yn_i^{\beta_a}, \quad j \in (n_i, 1] \tag{23}$$

Substituting for $c_{i,j}$ in (8) using (23) gives

$$c_{i,T} = (1-n_i)\left[\frac{1}{1-n_i}\int_{n_i}^{1} c_{i,j}^{1-\frac{1}{\phi}} dj\right]^{\frac{1}{1-\frac{1}{\phi}}} = (1-n_i)\, c_{i,n_i} = (1-n_i)\, yn_i^{\beta_a}. \tag{24}$$

Finally, on substituting for $c_{i,N}$ and $c_{i,T}$ from (21) and (24) in (6), we get

$$\begin{aligned} c_i &= \left[n_i\left(\frac{c_{i,N}}{n_i}\right)^{\frac{\phi-1}{\phi}} + (1-n_i)\left(\frac{c_{i,T}}{1-n_i}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \\ &= \left[\frac{y^{\frac{\phi-1}{\phi}} n_i^{\beta_a\frac{\phi-1}{\phi}+1}}{1+\beta_a\frac{\phi-1}{\phi}} + (1-n_i)\, y^{\frac{\phi-1}{\phi}} n_i^{\beta_a\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \\ &= yn_i^{\beta_a}\left[1-wn_i\right]^{\frac{\phi}{\phi-1}}. \end{aligned} \tag{25}$$

3.2 Characterizing the Set of Traded Goods, B

To further solve for the equilibrium, it is necessary to characterize the set B. We begin by determining how the marginal nontraded variety, n_i, varies with i for a (traded) good in B.

3.2.1 The Marginal Variety Frontier

Consumer's optimization over different goods i and $m \in B$ gives

$$\frac{c_i}{c_m} = \left[\frac{p_i}{p_m}\right]^{-\gamma}. \tag{26}$$

[6] As $c_{i,j} \leq y_{i,j}$, varieties $j \in (n_i, 1]$ are indeed exported.

With $m = 1$, using (19) and (25), one obtains[7]

$$n_i\left(n_1\right) \equiv i^{-\frac{\gamma\beta_c}{\beta_a}} \left[\frac{1 - wn_i}{1 - wn_1}\right]^{-\frac{1}{\beta_a}\frac{\phi-\gamma}{\phi-1}} n_1. \tag{27}$$

Given n_1, equation (27) solves for the share of nontraded varieties n_i for $i \in B$. In fact, it defines the *marginal variety frontier* in the space of goods. This frontier separates the nontraded and traded varieties of various goods. This analytical characterization is a result of the parameterization of trade costs as a function of i in (2).

This (analytical) characterization of the relationship between a sector's index and the index of its marginally nontraded variety captured in the marginal variety frontier is key to the simplification of the computation of the equilibrium of the model. In particular, as a result of this characterization, for example, to solve for the steady state, the computational algorithm needs to only make a guess for only one n_i, say, n_1. All other n_i's needed to solve for the steady state (or. more generally. a period's equilibrium) can be computed analytically using (27). Thus, as in Bergin and Glick, to solve for the steady state, the algorithm makes a guess on just one variable, despite the fact that our model features a continuum of sectors/goods and a continuum of varieties in each sector, whereas their's has just a continuum of goods.

The marginal variety frontier has the following intuitive property:

Proposition 1 *The marginal variety frontier is downward sloping, i.e.,*

$$\frac{dn_i}{di} < 0. \tag{28}$$

Proof. Isolating i on one side in (27) gives

$$i^{\frac{\gamma\beta_c}{\beta_a}} = \left[\frac{1 - wn_i}{1 - wn_1}\right]^{\frac{1}{\beta_a}\frac{\gamma-\phi}{\phi-1}} \frac{n_1}{n_i}. \tag{29}$$

Implicitly log differentiating (29) and simplifying gives

$$\frac{\gamma\beta_c}{\beta_a}\frac{n_i}{i}\frac{di}{dn_i} = -\left[1 - \frac{1}{\beta_a}\frac{\phi-\gamma}{\phi-1}\frac{wn_i}{1-wn_i}\right] = -\left[1 - \underbrace{\frac{\phi-\gamma}{\phi}}\,\underbrace{\frac{n_i}{1+\beta_a\frac{\phi-1}{\phi}(1-n_i)}}\right], \tag{30}$$

[7]In equilibrium, the economy at least exports the most abundant variety of the good with the least trade cost. In a more general case, there will exist at least some good m such that marginal nontraded condition holds.

where both terms inside square brackets on the left hand side that are marked with underbraces are positive and less than 1. Thus, right hand side of equation (30) is negative and so is dn_i/di. ■

Proposition 1 implies that the number of nontraded varieties falls, or equivalently the number of exported varieties rises, with i. This is quite intuitive as trade costs fall with i. One can, however, say more about the shape of the marginal variety frontier. The specific parametrization of trade cost in (2) implies that the *decrease* in trade costs becomes smaller as i increases. Thus, number of exported varieties rises slowly with i as next proposition shows.

Proposition 2 *The marginal variety frontier is convex, i.e.*

$$\frac{d^2 n_i}{di^2} > 0. \tag{31}$$

Proof. Differentiating both sides of (30) with respect to n_i and simplifying gives

$$\frac{\gamma\beta_c}{\beta_a}\left[\frac{n_i}{i}\frac{d^2 i}{dn_i^2} + \underbrace{\frac{1}{i}\frac{di}{dn_i}} - \underbrace{\frac{n_i}{i^2}\left(\frac{di}{dn_i}\right)^2}\right] = \frac{\phi-\gamma}{\phi}\frac{1+\beta_a\frac{\phi-1}{\phi}(2-n_i)}{\left[1+\beta_a\frac{\phi-1}{\phi}(1-n_i)\right]^2} > 0, \tag{32}$$

where both terms inside square brackets on left hand side that are marked with underbraces are negative. As right hand side is positive, the result follows immediately. ■

3.2.2 The Set of Traded Goods, B

The monotonicity of n_i in i proved in Proposition 1 allows an easy characterization of the set of traded goods, B.

Proposition 3 *The set of traded goods, $B = (\bar{\imath}, 1]$ where $\bar{\imath}$ solves (29) for $n_i = 1$, i.e.,*

$$\bar{\imath}\,(n_1) \equiv \left[\frac{1-w}{1-wn_1}\right]^{\frac{1}{\gamma\beta_c}\frac{\gamma-\phi}{\phi-1}} n_1^{\frac{\beta_a}{\gamma\beta_c}}, \tag{33}$$

and $0 < \bar{\imath} < 1$.

Proof. First, $\bar{\imath} > 0$ as good 0 has infinite trade cost. Second, $\bar{\imath} < 1$ because otherwise no good will be exported and this would violate the trade balance condition as with finite prices, consumers want to consume a positive amount of composite foreign good.

To see that interval $(\bar{\imath}, 1] \in B$, one may note that as i falls from 1 to $\bar{\imath}$ defined in (33), n_i rises from n_1 to 1. Thus, all goods in set $(\bar{\imath}, 1]$ have $n_i < 1$ *i.e.*, $(\bar{\imath}, 1] \in B$. Furthermore, B

cannot contain any other intervals: such intervals must be of type $(i_1, i_2]$, $i_1 \neq 0$ and $i_2 \leq \bar{\imath}$ which would need at least one more value of $i \neq \bar{\imath}$ to satisfy (27) for $n_i = 1$ which is not possible. Thus $B = [\bar{\imath}, 1]$ as claimed. ■

It may be pointed out that same ideas that obviated the need to consider conditions based on inequalities to determine the boundary between the traded and the nontraded varieties, have been, now, used in defining $\bar{\imath}$ as function of n_1 in (33). In particular, it is the assumption of a continuum of goods along with continuity of trade costs over this continuum that ensures that $\bar{\imath}$ is defined by an equality as in Bergin and Glick (2009). However, they only consider heterogeneity in one dimension at a time.

3.2.3 The Marginal Fully Nontraded Good

Like the marginal nontraded variety, n_i, the good $\bar{\imath}$ defined by (33) is also special. We label good $\bar{\imath}$ as the *marginal fully nontraded* good. It is the marginal fully nontraded good in the sense that the marginal nontraded variety condition holds with $n_i = 1$. It plays the same role across goods as variety n_i plays in linking the price of traded and nontraded varieties for a good $i \geq \bar{\imath}$.

3.3 Solving for the Prices of Nontraded Goods

The price of the marginal fully nontraded good is given by

$$p_{\bar{\imath}} = p_{\bar{\imath},N} = \frac{p^* \bar{\imath}^{\beta_c}}{\alpha} \left[1 + \beta_a \frac{\phi - 1}{\phi}\right]^{\frac{1}{\phi - 1}}, \tag{34}$$

and for any other fully nontraded good $i < \bar{\imath}$, consumer optimization implies

$$p_i = \left[\frac{c_i}{c_{\bar{\imath}}}\right]^{-\frac{1}{\gamma}} p_{\bar{\imath}} = p_{\bar{\imath}}, \quad i \leq \bar{\imath}, \tag{35}$$

where last equality follows from using (25) for aggregate consumption index for goods i and $\bar{\imath}$ noting that $n_i = n_{\bar{\imath}} = 1$. It also follows that prices of different varieties of a fully nontraded good are same as that of the corresponding variety of the marginally nontraded good $\bar{\imath}$.[8]

[8]For this, use Eq. (14) to substitute for $p_{i,j}$ in terms of $p_{i,1}$ in Eq. (12) and integrate it to obtain an expression of p_i in terms of $p_{i,1}$ and compare it with expression for p_i from (34) and (35) to conclude that $p_{i,1}$ is independent of i for $i < \bar{\imath}$. Eq (14) then implies that $p_{i,j}$ is also independent of i for $i < \bar{\imath}$.

3.4 The Final Step

Equations (19), (25), and (35) express all prices and consumption choices in terms of n_i and $\bar{\imath}$. Both n_i and $\bar{\imath}$, in turn, are function of n_1 by virtue of equations (27) and (33). Thus, we only need to know n_1 to solve for the equilibrium of the model for any period or in the steady state. The value of n_1, the share of non-traded varieties for good 1, is determined by the external balance condition or the budget constraint. The exact details depend on the nature of the analysis.

3.4.1 Steady State Analysis

In the steady state case, with current account in balance, we have

$$p_H c_H \equiv \int_0^1 p_i c_i di = \theta \int_0^1 p_i y_i di \equiv \theta p_H y_H. \tag{36}$$

That is, the expenditure on domestic goods is a fraction θ of the total income in accordance with preferences in (4).

In order to be able to use (36) to solve for n_1, we need to express all variable therein in terms of n_1. Towards this end, note that, for $i \geq \bar{\imath}$,

$$p_i y_i = \frac{p^* i^{\beta_c}}{\alpha} y \left(\int_0^{n_i} \left(\frac{j}{n_i} \right)^{-\frac{\beta_a}{\phi}} j^{\beta_a} dj + \int_{n_i}^1 j^{\beta_a} dj \right) = \frac{p^* i^{\beta_c}}{\alpha} \frac{y}{1+\beta_a} \left[1 + \frac{w}{\phi - 1} n_i^{1+\beta_a} \right], \tag{37}$$

and for $i \leq \bar{\imath}$,

$$p_i y_i = \frac{p^* \bar{\imath}^{\beta_c}}{\alpha} \frac{y}{1+\beta_a} \left[1 + \frac{w}{\phi - 1} \right]. \tag{38}$$

Also, from (19), (25),and (30), one sees that

$$\begin{aligned} p_i c_i &= \frac{p^* i^{\beta_c}}{\alpha} y n_i^{\beta_a} \left[1 - w n_i \right], \qquad i \geq \bar{\imath} && (39) \\ &= \frac{p^* \bar{\imath}^{\beta_c}}{\alpha} y \left[1 - w \right], \qquad\qquad i \leq \bar{\imath} && (40) \end{aligned}$$

Substituting for $p_i c_i$ and $p_i y_i$ from (37-40) into (36) and noting (1) that for $i \geq \bar{\imath}$, (27) gives

n_i as function of i and n_1, and (2) that (33) gives $\bar{\imath}$ as a function of n_1, one obtains[9]

$$\int_0^{\bar{\imath}(n_1)} \frac{p^*\bar{\imath}(n_1)^{\beta_c}}{\alpha} y\left[1-w\right] di + \int_{\bar{\imath}(n_1)}^1 \frac{p^* i^{\beta_c}}{\alpha} y n_i(i,n_1)^{\beta_a}\left[1-w n_i(i,n_1)\right] di$$
$$= \theta \int_0^{\bar{\imath}(n_1)} \frac{p^*\bar{\imath}(n_1)^{\beta_c}}{\alpha}\frac{y}{1+\beta_a}\left[1+\frac{w}{\phi-1}\right] di + \theta \int_{\bar{\imath}(n_1)}^1 \frac{p^* i^{\beta_c}}{\alpha}\frac{y}{1+\beta_a}\left[1+\frac{w}{\phi-1} n_i(i,n_1)^{1+\beta_a}\right] di, \tag{41}$$

which can be numerically solved for n_1. Like the corresponding equation in Bergin and Glick, it is one equation in one unknown, albeit more complicated. This reduction of computation of equilibrium (specifically steady state in this case) to one equation in one unknown has relied solely on the analytical characterization of marginal variety frontier in (27) as (33) itself was derived from (27).

3.4.2 The Two Period Model

Before proceeding to numerical computation of the model, it may be mentioned that the facts that the prices of traded and nontraded varieties and goods by linked by equalities hold irrespective of whether we look at a static or a dynamic model. Hence, it is still possible to reduce of the computation of equilibrium to solving one equation in one unknown. The differences are only in details.

In the case of a dynamic model, the final step imposes intertemporal budget constraint and intertemporal efficiency in consumption. For example, consider a two period model with utility function

$$\delta u\left(c^1\right) + u\left(c^2\right), \tag{42}$$

as in Bergin and Glick (2009), where c^k is the aggregate consumption index for period $k = 1, 2$ and δ is a demand shock. The intertemporal budget constraint in this case is

$$p^1 c^1 + \frac{1}{1+r} p^2 c^2 = p_H^1 y_H^1 + \frac{1}{1+r} p_H^2 y_H^2, \tag{43}$$

where r is the world interest rate and p^k, and $p_H^k y_H^k$ are the aggregate price level and value of the endowment of home goods for period $k = 1, 2$.

Consumer optimization yields the Euler equation

$$u'\left(c^1\right) = \frac{1+r}{\delta} u'\left(c^2\right). \tag{44}$$

[9] For $i < \bar{\imath}$, $n_i = 1$.

In addition, it also yields that a fraction θ of domestic consumption expenditure falls on domestic goods, i.e.,

$$\int_0^1 p_i^k c_i^k di = \theta p^k c^k, \quad k = 1, 2. \tag{45-46}$$

Using (45) and (46) to eliminate c^k, $k = 1, 2$ from (43) and (44) one obtains two equations that can be solved for n_1^k, $k = 1, 2$, the share of good 1 that is nontraded in period k. The computation of remaining equilibrium follows in the manner described for the steady state case. Since the essential elements of computational simplification are same, in what follows, we restrict attention to the steady state analysis.

4 Calibration and Numerical Simulations

The numerical simulations further highlight the advantages of incorporating heterogeneity in two dimensions. Certain problems that dog the calibration, when considering heterogeneity in one dimension, get solved in a natural way. For example, Bergin and Glick (2009) order varieties of all goods on a single continuum based on trade costs.[10] Yet, for their quantitative analysis, this poses a problem, as empirically, it is the case that elasticity of substitution between varieties of same good (ϕ) is different (and much higher) than the elasticity of substitution between two varieties of different goods.[11] With goods and varieties ordered on a double continuum, this problem does not arise in our model and we have a more accurate calibration of the model by virtue of being able to distinguish between the elasticity of substitution across goods and across varieties of a good.

Given the similarity, the benchmark case for numerical simulations is essentially same as that in Bergin and Glick with ϕ set at 10, but the elasticity of substitution among goods, γ, is given a lower and more plausible value of 2. In addition, the new parameter of the model, β_a, is given a value of 4 so that endowment of different varieties of a good captures empirically plausible differences in productivity of firms and skewness of the firm size distribution in a sector.[12] In particular, it implies that the output of the largest firm in the sector is 5 times the average output. Put differently, 66.9% of the firms in the sector produce output less than

[10] They cite empirical evidence on heterogeneity of trade costs both within and across sectors for this purpose.

[11] One way to avoid this problem will be to ignore heterogeneity of trade costs between varieties of same good as in this paper. With this interpretation, their continuum ranks goods. But in that case, their choice of elasticity of substitution of 10 in the base case is quite high. A value of 10 is more reasonable for elasticity of substitution among different varieties of the same good, perhaps a value of 2 may be empirically more plausible for elasticity of substitution between different goods. However, in that case, the relative price of nontraded good becomes more volatile when compared to empirical evidence unless the elasticity of transport costs is adjusted upwards.

[12] Recall that differences in endowment are similar to differences in productivity in a model with production.

Endowments		
$\beta_a = 4$	$y = 1$	
Trade costs		
$\beta_c = 1.5$	$\alpha = 1$	
Preferences		
$\phi = 10$	$\gamma = 2$	$\theta = .5$
World prices		
$p_F = 1$	$p^* = 1$	

Table 1: Benchmark Parametrization of the Model.

the firm producing the average output and 13% of firms produce half the industry output. The calibrated values of the parameters are collected in Table 1. In particular, note that the consumption share of imported goods (θ) is .50. Further, β_c, the elasticity of trade costs with respect to i takes a value of 1.5.

Figure 1 shows the equilibrium quantities and prices for the calibrated model. The endowment profile is same for all goods. However, as trade costs are lower for goods with high i, the economy exports relatively abundant varieties of these goods. The consumption profile shows the economy exports most of the goods with lower trade costs to pay for imports of the composite foreign good. This pattern of exports is reflected in domestic prices. The domestic prices of varieties of goods fall with j as endowment of varieties rises, but, at some point prices fall so much that it is becomes profitable to export those varieties. This happens earlier for goods with lower trade costs. This can be easily seen in the profile of exports in Figure 1, where the marginal variety frontier separating exported and nontraded varieties is most clearly visible.

Figure 2 shows the marginal variety frontier in the goods space. It is convex and downward sloping as proved earlier. More importantly, even for the most-traded good ($i = 1$), 42.35% varieties are not traded, and among the goods, 44.37% are fully nontraded (as $\bar{\imath} = .4437$.) Overall 77.70% of the firms do not export their products.

4.1 Implications of Sectoral and Firm-Level Heterogeneity for Deviations of Domestic Prices from World Prices

Models with heterogeneity either in trade costs or in productivity make contrasting predictions about the deviation of the domestic price from the world price for the traded and nontraded goods.

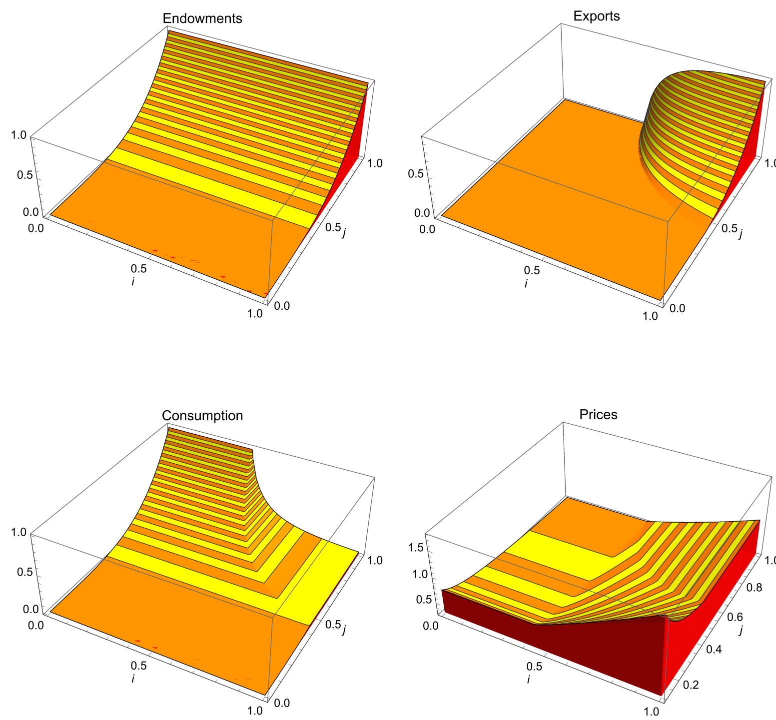


Figure 1: Quantities and Prices in the Calibrated Model.

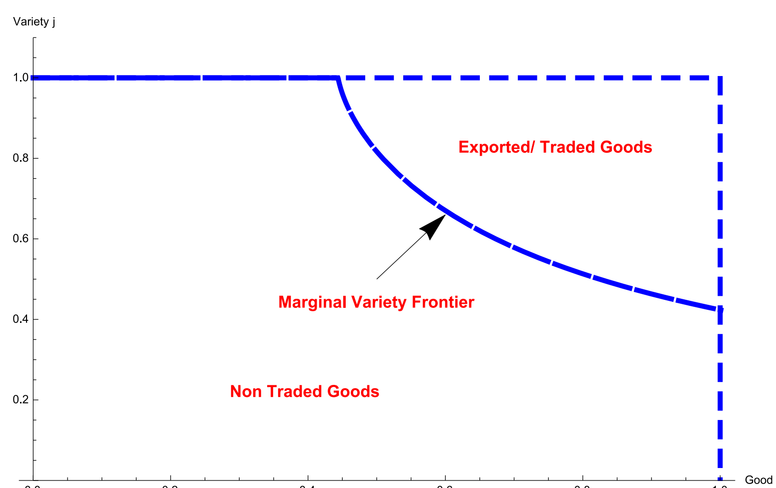


Figure 2: The Marginal Variety Frontier.

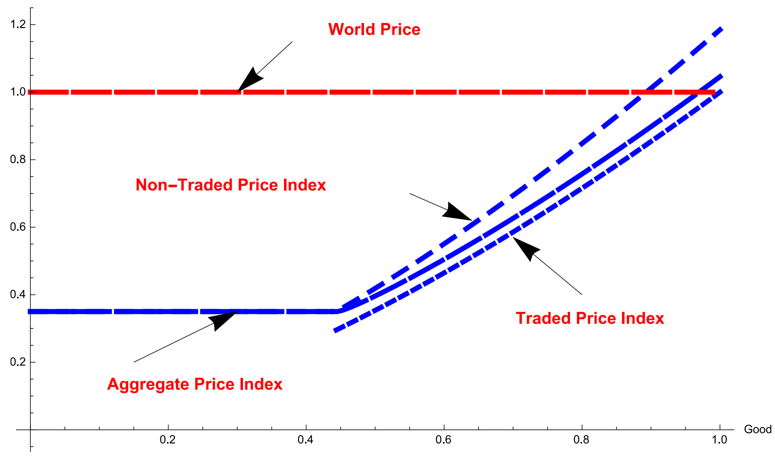


Figure 3: Price Indices of Traded and Nontraded Varieties of Various Goods.

In the model with heterogeneity only in productivity a la Dornbusch *et. al.* (1977), the deviation from world price is smaller for the prices of the nontraded goods than for those of the traded goods. The domestic price of traded goods differs from their world price by the trade costs whereas those of nontraded goods differs by less making them unprofitable to export. On the other hand, in the models with heterogeneity in trade costs such as Bergin and Glick (2009), the reverse is true.

Crucini, Telmer, and Zachariadis (2005) show that deviations from law of one price are larger for the nontraded goods as suggested by the models with heterogeneous trade costs. More specifically, they find that measures of cross-sectional price dispersion are negatively related to the tradability of the good, and positively related to the share of non-traded inputs required to produce the good.

Our model is consistent with the empirical results in Crucini, Telmer, and Zachariadis (2005). For this look at the Figure 3, which shows the aggregate price indices of various goods (p_i) as well as price indices for the traded ($p_{i,T}$) and the nontraded ($p_{i,N}$) varieties. The horizontal line shows the world price of all varieties of all goods. For the economy as a whole, the deviations from world price are much higher for nontraded goods.

However, our model, with heterogeneity along both dimensions, goes further. It also provides an explanation for why the predictions of the model with heterogeneous productivity are not borne out at the economy-wide scale. Since the heterogeneity in productivity is a sector level phenomenon, its predictions hold only at the sector level: For a particular good, the deviations from world price are lesser for nontraded varieties than for the traded varieties. Indeed, this is a testable prediction of the model. Thus, the apparently conflicting predictions of models with one-dimensional heterogeneity (along different dimensions) appear entirely consistent in our more general model.

5 Reduction in Trade Costs and Nontradability

This section examines sectoral pattern of changes in nontradability in response to changes in trade costs. The trade costs that the small open economy faces may fall as a result of signing a free-trade agreement with a large country or joining a regional trade bloc. The reduction in trade costs is modeled as a fall in β_c in spirit of the analysis in Bergin and Glick (2009) who analyze the volatility of relative price of nontraded goods for different values of β_c. As we shall see, nontradability in different sectors responds differently to the reduction in trade costs. It would not be possible to arrive at this result by combining the predictions of the models that incorporate heterogeneity in one dimension at a time. Incorporating both sectoral heterogeneity in trade costs and firm-level heterogeneity in productivity in the same model turns out to be essential.

5.1 Changes in Sectoral Nontradability in Presence of Sectoral and Firm-Level Heterogeneity

While the model is complex, as it simultaneously incorporates heterogeneity in two dimensions, it is possible to show that:

Lemma 4 *Given n_1, the marginal variety frontier becomes flatter as trade costs fall, i.e., as β_c falls.*

Proof. For $i = 1$, equation (30) yields

$$\left.\frac{dn_i}{di}\right|_{i=1} = -\frac{\gamma n_1}{\beta_a}\left[1 - \frac{\phi - \gamma}{\phi}\frac{n_1}{1 + \beta_a\frac{\phi-1}{\phi}(1 - n_1)}\right]^{-1}\beta_c, \tag{47}$$

which shows that the marginal variety frontier is flatter at $i = 1$ for a lower β_c. ■

Combining the fact in Lemma 4 with the results, from Propositions 1 and 2, that marginal variety frontier is downward sloping and convex, immediately yields the following result:

Corollary 5 *Given n_1,*

$$\frac{\partial n_i}{\partial \beta_c} < 0, \tag{48}$$

$$\frac{\partial \bar{\imath}}{\partial \beta_c} < 0. \tag{49}$$

However, the direction of movement of n_1, and that of the marginal variety frontier, can not be established analytically. Following three outcomes are potentially possible: (i) n_1

rises by so much that despite new marginal variety frontier being flatter, it lies everywhere above the old one; (ii) n_1 rises but not so much thus while new marginal variety frontier starts above the old one at $i = 1$, but being flatter, it then cross the old frontier from above as i falls; or (iii) n_1 falls and the new marginal variety frontier is everywhere below the old one.

It is possible to rule out outcome (i) through very intuitive heuristic arguments. The hint to implausibility of (i) is provided by its questionable prediction that $\bar{\imath}$ rises so that more goods become *non*tradable with fall in trade costs. We begin by noting that trade costs depress the domestic prices of Home goods relative to world price (see Figure 3). Thus, a fall in trade costs raises Home GDP. With Cobb-Douglas preferences, therefore, optimal consumption allocation requires that the nominal value of exports must rise and maintain the same ratio to GDP as before. While reduction in trade costs increases the price received by Home for its exports, the value of initial exports rises less than proportionally when compared to the GDP. This happens because the price of lesser traded goods (and non-traded in particular) rises much more and the weight of exported varieties in those goods is smaller. As the quantity of exports also falls, clearly the value of exports as share of GDP declines, contradicting optimality of consumption allocation between Home and Foreign goods.

With outcome (i) excluded, we now have the intuitive prediction that $\bar{\imath}$ must fall making more goods tradable. However, to proceed beyond that, it is necessary to turn to numerical simulations. Surprisingly, despite analytical ambiguity vis-a-vis outcomes (ii) and (iii), numerical results for a wide variety of plausible parameter values and preference and supply-side assumptions, show, without exception, that n_1 actually rises with the fall in trade costs.

The benchmark case for numerical simulation considers the effect of reduction in β_c from 1.5 to 1. This reduction in β_c reduces both average trade costs for the exports and the dispersion of trade costs across the exports. The effect of fall in β_c on endogenous nontradability for various goods (i, j) is shown in Figure 4. As expected more goods become tradable as a result of reduced trade costs. In particular, $\bar{\imath}$ falls from .444 to .353. However, despite a reduction in trade costs, not every traded sector (*i.e.* sector with some traded varieties to begin with) experiences an increase in the number of varieties that are traded. In fact, for sectors with least trade cost, both the number of varieties traded and the export of goods actually *falls*. This happens in 53.48% of the sectors that previously exported some varieties. However, the number of varieties traded and exports for the economy rise because this fall is more than compensated by the increases coming from other traded goods and the goods that turn from being nontraded to traded. Thus, as expected reduction in trade costs indeed increases overall trade, both in terms of volume of exports and number of firms exporting, but its impact varies by sector.

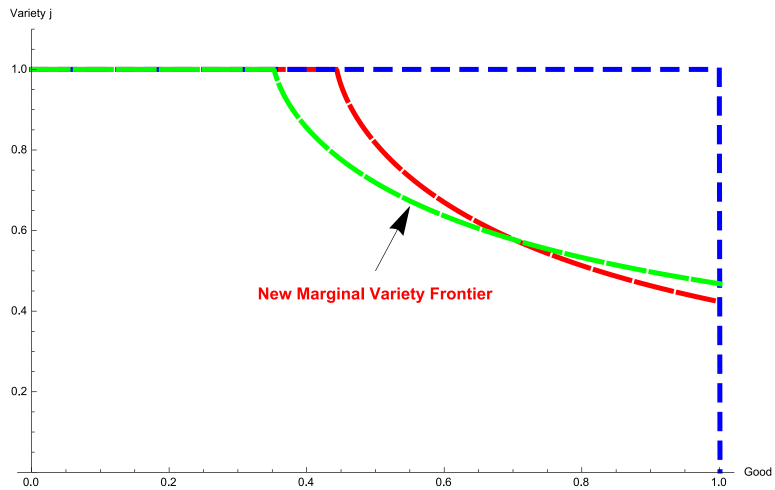


Figure 4: Reduction in Trade Costs and Sectoral Changes in Nontradability.

Some intuition for the seemingly odd outcome that number of varieties for most tradable goods actually falls can be, however, provided. The reform reduces both the level and the dispersion of trade costs across goods. With reduced dispersion, it becomes optimal to save on the varieties with low j for goods with high i as their domestic supply is low, and instead, to meet its import needs, export more of varieties with high j for goods with low i that are in greater supply.

It is not possible to obtain this result in a model where heterogeneity in both dimensions is not present simultaneously.

Suppose, one ascertains the effect on tradability of various goods by ignoring firm-level heterogeneity in productivity. In that set up, a reduction in β_c will imply that more goods become tradable as in our model. While this is a fairly general and robust result, specifically, this is the outcome that obtains in Bergin and Glick (2009) which only considers sectoral variations in trade costs.

Similarly, suppose to ascertain the effect on tradability of various varieties in a particular sector, one ignores heterogeneity in trade costs and only consider heterogeneity in productivity. The reduction in β_c in our model will correspond to reduction in the *level* of trade costs in this one sector setup. Such a reduction in trade cost will increase the number of varieties that are traded. For example, this is the outcome that obtains in Melitz (2003) which only allow firm-level variations in productivity, when either (iceberg) trade costs or fixed costs of entry in export markets are reduced.

If one tries to piece together the results from these two separate experiments and assert that a decrease in trade costs (a reduction in β_c) will both make more goods tradable and more varieties of traded goods tradable, it would clearly be incorrect. In terms of Figure 4, this would imply new marginal variety frontier lying everywhere below the old one, a case that our analysis has ruled out.

5.2 Sensitivity Analysis

Given the intuition suggested above, it is not surprising that the result is quite robust to the choice of parameter values. Table 2 presents the results of sensitivity analysis that support this claim. First column lists the new values of parameters that are changed. Next two columns report the values of n_1 and $\bar{\imath}$ before reduction in β_c. Next two column report change in n_1 as percent of total varieties and nontraded varieties when β_c falls whereas the following column shows the per cent change in $1-n_1$.[13] The i^* in the next column is the good for which number of traded varieties remains unchanged. This is the value of i corresponding to the intersection of the old and new marginal variety frontiers (see Figure 4). In the last column, we report the percent of initial traded goods for which number of traded varieties falls.

A first look at Table 2 shows that the number of traded varieties as percent of initially traded varieties ($\widehat{1-n_1}$) for most traded good falls by $5-10\%$. Not only that, anywhere between $45-75\%$ of initially traded sectors experience a decrease in the number of traded varieties. In all these alternatives, the initial equilibrium also changes in intuitive relative to the benchmark case.

Second and third rows of Table 2 confirm the robustness of the result to changes in firm size or profile of endowment. Furthermore, with a more skewed endowment for $\beta_a = 6$, n_1 is higher than the base case because agents have very little relative endowment of lower j varieties and they want to consume them rather than export. Next two sets of rows demonstrate robustness to different combinations of the elasticities of substitution between goods and that between varieties. For the initial equilibrium, the general pattern that emerges is that with larger values of elasticities the economy exports more varieties of highly traded goods as substitution in consumption is easy. In the last two rows, we vary the consumption share of the composite imported foreign good. The initial values of n_1 and $\bar{\imath}$ change in intuitive ways with θ and, pursuant to fall in β_c, n_1 rises again, with more than 50% of traded goods still experiencing a decline in the number of traded varieties.

The outcome is also robust to the initial level of trade costs (β^c) and the extent of reduction in trade costs. The results are shown in Table 3. With a larger percentage reduction in β_c (from β^H to β^L) the effects identified above get stronger as comparison of results within any of the panels of Table 3 shows. The initial equilibrium also changes in

[13]Specifically, we have $\widehat{1-n_1} = (\Delta(1-n_1))/(1-n_1) = -\Delta n_1/(1-n_1)$.

	n_1	$\bar{\imath}$	Δn_1	$\widehat{n_1}$	$\widehat{1-n_1}$	i^*	$\frac{1-i^*}{1-\bar{\imath}}100$
Base case	.4235	.4437	4.47	10.54	−7.74	.7025	53.48
$\beta_a = 6$	.5544	.4420	3.93	7.09	−8.82	.7022	53.38
$\beta_a = 2$	.2016	.4477	4.23	20.95	−5.29	.7031	53.75
$\gamma = \phi = 2$	.4415	.3361	4.25	9.63	−7.61	.6922	46.37
$\gamma = \phi = 10$	.1048	.5480	6.36	60.66	−7.10	.6843	69.83
$\gamma = 2, \phi = 20$	.4215	.4562	4.47	10.62	−7.73	.7034	54.54
$\gamma = 5, \phi = 20$	.2367	.5351	6.38	26.96	−8.36	.6965	65.28
$\gamma = 10, \phi = 20$	.0961	.5642	5.96	61.98	−6.59	.6836	72.60
$\theta = .25$	.3089	.3023	4.30	13.94	−6.23	.6178	54.78
$\theta = .75$	.5476	.5965	4.12	7.52	−9.10	.7879	52.58

Table 2: Sensitivity Analysis for Parameter Values.

intuitive ways with the changes in level of trade costs. For example, when β^H rises from 1.5 to 3, to offset the increase in trade costs of goods with low i, the economy now exports a larger fraction of varieties of more tradable goods with higher i.

	n_1	$\bar{\imath}$	Δn_1	$\widehat{n_1}$	$\widehat{1-n_1}$	i^*	$\frac{1-i^*}{1-\bar{\imath}}100$
Base case	.4235	.4437	4.47	10.54	−7.74	.7025	53.48
$\beta_c^H = 1.5; \beta_c^L = .75$	.4235	.4437	7.44	17.57	−12.91	.6855	56.53
$\beta_c^H = 1.5; \beta_c^L = .50$	.4235	.4437	11.30	26.67	−19.59	.6638	60.43
$\beta_c^H = 3; \beta_c^L = 2$	.3433	.5868	4.72	13.75	−7.19	.7908	50.62
$\beta_c^H = 3; \beta_c^L = 1$	.3433	.5868	12.49	36.37	−19.01	.7571	58.80
$\beta_c^H = .75; \beta_c^L = .50$	.4979	.2872	3.86	7.74	−7.68	.6028	55.73
$\beta_c^H = .75; \beta_c^L = .25$	.4979	.2872	9.35	18.77	−18.61	.5630	61.31

Table 3: Sensitivity Analysis for Level and Change in Trade Costs.

5.3 More General Preferences

The model assumes Cobb-Douglas preferences over the composite foreign and home good. This forces agent's to spend a constant fraction of their income on the imported good. The effect of the reduction of trade costs on nontradability is likely to be depend on the elasticity of substitution in consumption. The results in Table 4, accordingly, allow for the CES preferences

$$c = \left[\theta c_H^{1-\frac{1}{\kappa}} + (1-\theta)\, c_F^{1-\frac{1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}}. \tag{50}$$

Following the reduction in β_c, the move toward nontradability in more tradable sectors becomes stronger if elasticity of substitution (κ) between home and foreign goods falls as comparison of first two rows (fifth column) in Table 4 reveals. Recall, $\kappa = 1$ for the base case. This is expected. To see this, recall, the prices of nontraded goods at economy level is lower due to trade costs (see Figure 3). The reduction in trade costs raises the prices of domestic goods by raising the price of the nontraded goods. With domestic goods becoming expensive, smaller share of income is spent on foreign composite good for $\kappa < 1$. This leads to a larger rise in n_1 when β_c is reduced. In last two rows, θ is adjusted so that, the consumption share of foreign good remains at 50% in the initial equilibrium as in the base case. The results for changes in nontradability, however, turn out be very similar.

	θ	n_1	$\bar{\imath}$	Δn_1	$\widehat{n_1}$	$\widehat{1-n_1}$	i^*	$\frac{1-i^*}{1-\bar{\imath}}100$
Base case	.5	.4235	.4437	4.47	10.54	−7.74	.7025	53.48
$\kappa = .5$	.5	.3731	.3811	5.86	15.72	−9.36	.5918	65.95
$\kappa = 1.5$	.5	.4623	.4918	3.55	7.68	−6.61	.7737	44.52
$\kappa = .5$	.5968	.4235	.4437	5.69	10.54	−7.74	.6425	64.27
$\kappa = 1.5$	.4033	.4235	.4437	3.49	8.24	−6.05	.7561	43.84

Table 4: Trade Costs and More General Preferences.

5.4 Production

It is straightforward to introduce production in this economy. Let the production function for good (i, j) be

$$y_{i,j} = A_{i,j} \left(l_{i,j}\right)^a, \quad 0 \leq a \leq 1. \tag{51}$$

Now, the heterogeneity of endowment is replaced by heterogeneity in productivity. As with endowments earlier, the economy has a higher productivity of the variety with higher index j

$$A_{i,j} = A j^{\beta_a}. \tag{52}$$

The other details for the model, including that for the computation of equilibrium, are very similar to the endowment economy and hence, are relegated to Appendix A. From computational perspective, as before, there is no curse of dimensionality and concomitant increase in complexity. One still uses the external balance condition $(A.25)$ to solve for equilibrium value of n_1. However, at each step, given a value of n_1, one has to also solve $(A.20)$ for the wage rate, W.

The results for economy with production for a reduction in β_c from 1.5 to 1 are shown in Table 5. First two rows compare the production economy with the endowment economy for same β_a. The production economy has a greater reduction in traded varieties $(\widehat{1-n_1})$. Also, a larger number of initial traded goods witness lesser varieties being exported. However, same β_a implies larger dispersion in supply of different varieties for production economy than the

	β_a	θ	n_1	$\bar{\imath}$	Δn_1	$\widehat{n_1}$	$\widehat{1-n_1}$	i^*	$\frac{1-i^*}{1-\bar{\imath}}100$
$\boldsymbol{\kappa = 1.0}$									
Endowment	4	.5	.4235	.4437	4.47	10.54	−7.74	.7025	53.48
$a = .5$	4	.5	.6210	.5485	4.67	7.51	−12.31	.7175	62.56
$a = .5$	2.2	.5	.4345	.5511	4.02	9.26	−7.11	.7940	45.90
$a = .8$	1.12	.5	.4732	.6973	3.41	7.21	−6.47	.8895	36.49
$\boldsymbol{\kappa = .5}$									
Endowment	4	.5968	.4235	.4437	5.69	13.42	−9.86	.6425	64.27
$a = .5$	2.2	.5712	.4345	.5511	4.75	10.95	−8.41	.7630	52.79
$a = .8$	1.12	.5395	.4732	.6972	3.72	7.86	−7.06	.8805	39.48

Table 5: Trade Costs in an Economy with Production.

endowment economy. In third row, β_a is, therefore, reduced from 4 to 2.2 so that initial distribution of supply of varieties for a good is same in the endowment and the production economy. In the fourth row, a is increased so that decreasing returns in production are weaker. While, in these cases, the results get weaker compared to endowment case, but still traded varieties fall for the most-tradable good and at least a third of earlier traded goods have less varieties exported. The results are similar for a lower elasticity of substitution between foreign and domestic composite good.

6 A Two Country Set up

This section considers a two-country model and shows that the insight concerning avoidance of curse of dimensionality and computational complexity illustrated so far for small-open economy extends in a straightforward manner to this case. To illustrate the technique and outline the details, we consider two countries, Home and Foreign, that are identical in all respects except that their endowment of different varieties of a good is anti-symmetric so that

$$y_{i,j} = y\left(\frac{1+j}{\delta}\right)^{\beta_a}, \tag{53}$$

$$y^*_{i,j} = y\left(\frac{2-j}{\delta}\right)^{\beta_a}. \tag{54}$$

with $y > 0$, $\beta_a > 0$, and $\delta > 0$.

The other details of the model, including that of solving the model, are similar to the small-open economy case, and hence, are relegated to Appendix B. Here we only point out some salient features. In contrast to the small-open economy case, now, for every sector there exists a *marginal imported variety*, n_i, such that varieties $j \in [0, n_i)$ are imported by

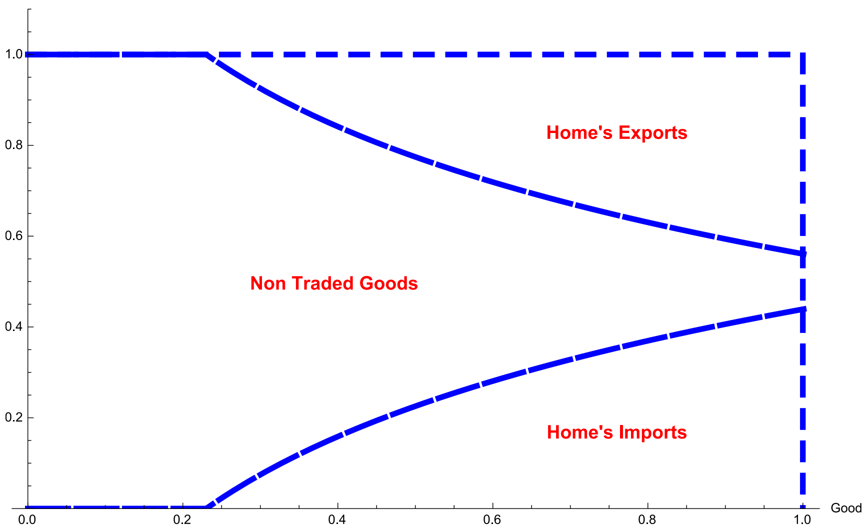
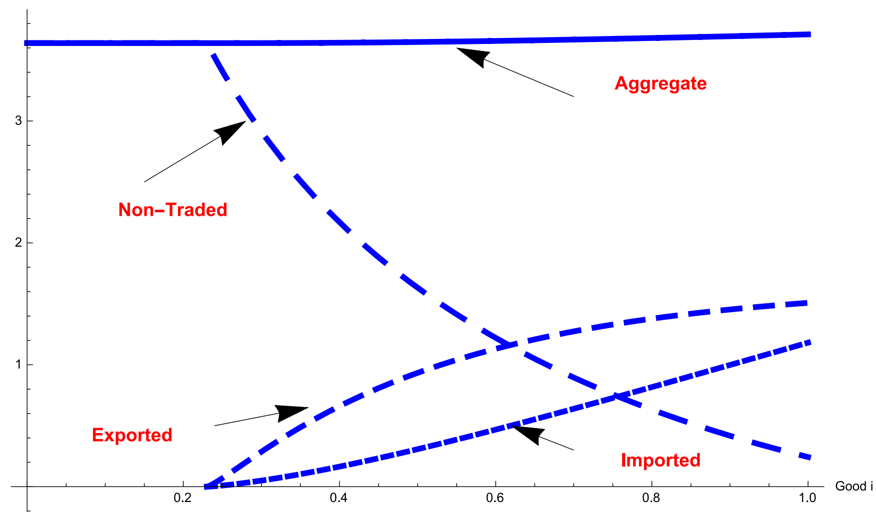


Figure 5: Pattern of Trade in Two Country Model.

Home. Similarly, there exists a *marginal exported variety,* which by symmetry is $1-n_i$, such that varieties $j \in (1-n_i, 1]$ are exported by Home. The rest of the varieties $[n_i, 1-n_i]$ are nontraded. The assumed symmetry also implies that, as in case of small open economy, there is a *marginal fully nontraded good*, $\bar{\imath}$, with $n_{\bar{\imath}} = 0$.

We would like to highlight the fact that, as in the case of a small-open economy, to solve for the entire equilibrium, one needs to solve only one equation (B.27) in one unknown, n_1. However, if the two countries were dissimilar in other respects, besides having anti-symmetric endowments, one will need to solve two equations in two unknowns, the marginal imported and exported variety of good 1 for Home, n_1^m and n_1^x. In the symmetric case discussed here, $n_1 = n_1^m = 1 - n_1^x$.

For illustrative purposes, Figure 5 shows the pattern of trade for the following calibration of the model: $\gamma = 2$, $\phi = 5$, $\beta_a = 3$, $\beta_c = .25$, $\alpha = 1.05$, $y = \delta = 1$. Figure 6 (top graph) shows consumption of imported, nontraded , and exported varieties of good for various goods for Home. It shows that even though number of varieties exported and imported are same due to assumed symmetry, yet the consumption of imported varieties of any good is less than that of the exported varieties. This is due to differences in their prices, both due to differences in endowments and trade costs. In the bottom graph of Figure 6, one can see that the price of imported varieties falls with i as the trade cost falls. On the other hand, the price of export varieties rises as they are exported in larger quantity with fall in trade costs. The price index of a good and its nontraded varieties shows little variation in this case shown.



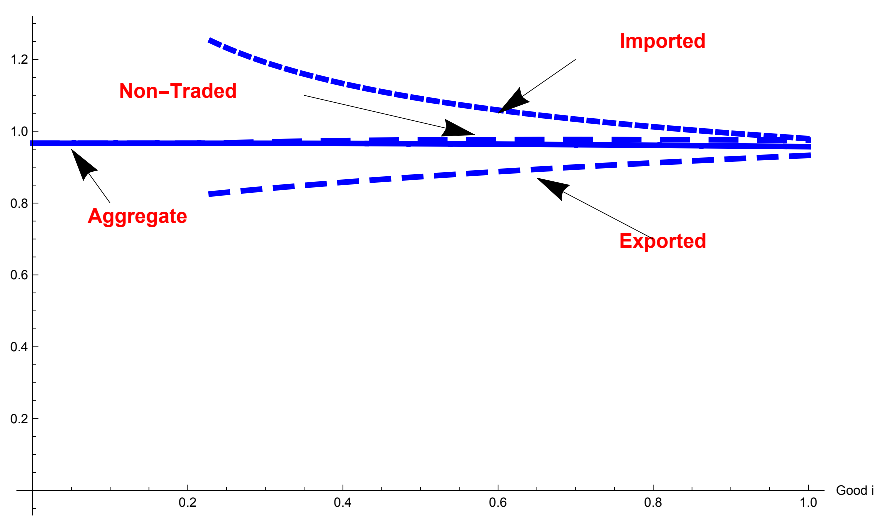


Figure 6: Consumption (Top) and Price (Bottom) Indices for Imported, Exported, and Non-Traded Varieties.

7 Concluding Remarks

The availability of detailed firm-level economy-wide data has significantly transformed the field of international trade over past decade. It has also motivated the development of theoretical models in which heterogeneity plays a vital role. These microfoundations have also been incorporated into models of international macroeconomics. Some of the predictions of these models have been tested on data and with success. But, much more can be done to test the relevance of or improve the predictions of these theories by incorporating heterogeneity both across sectors and firms. For example, a theory of firm selection into export market will not only have prediction about how this selection margin is affected when tariffs are reduced but also about how this effect varies across different sectors depending on differences in factor intensity, trade costs, or technology.

These sectoral variations may be difficult to predict using simpler models. As the paper shows, in a model that includes, perhaps the most important sources of sectoral heterogeneity (trade costs) and firm-level heterogeneity (productivity), the predictions of a simplified approach incorporating heterogeneity in one dimension at a time are overturned. The model also, in a natural way, reconciles the contrasting, and apparently contradicting, predictions of models with heterogeneity in only trade costs or productivity about the differences in the deviation of domestic price from the world price for the traded and nontraded goods.

The paper demonstrates that building models necessary for such analysis and solving them numerically can be simplified by introducing heterogeneity along another dimension over a continuum rather than over a countable or finite set. While such an extension increases computational time to some extent, it does not increase the dimensionality of the problem.

Appendix A. A Small Open Economy with Production

As shown by Bergin and Glick (2009) for heterogeneity in one dimension, introducing production does not involve any additional issues. Following them, the production function for good (i, j) is

$$y_{i,j} = A_{i,j} \left(l_{i,j}\right)^{a}, \quad 0 \leq a \leq 1. \tag{A.1}$$

The economy has a higher productivity of the variety with higher index j

$$A_{i,j} = A j^{\beta_a}. \tag{A.2}$$

With perfect competition, price equals marginal cost for each good (i, j) and wage (W) is equalized across goods and varieties, which gives

$$p_{i,j} = \frac{W}{a y_{i,j}/l_{i,j}} = \frac{W}{a \left(A_{i,j}\right)^{1/a}} \left(y_{i,j}\right)^{1/e}, \tag{A.3}$$

$$y_{i,j} = \left(\frac{a \left(A_{i,j}\right)^{1/a}}{W} p_{i,j} \right)^{e}. \tag{A.4}$$

where $e = a/\left(1-a\right)$ is the elasticity of output with respect to price. Thus, for any two varieties j and k of a good i, we have

$$\frac{y_{i,j}}{y_{i,k}} = \left(\frac{p_{i,j}}{p_{i,k}}\right)^{e} \left(\frac{A_{i,j}}{A_{i,k}}\right)^{1+e}, \tag{A.5}$$

On the other hand, relating outputs and prices by consumer optimization gives

$$\frac{p_{i,j}}{p_{i,k}} = \left[\frac{y_{i,j}}{y_{i,k}}\right]^{-\frac{1}{\phi}}. \tag{A.6}$$

Equations (A.5) and (A.6) together yield

$$\frac{p_{i,j}}{p_{i,k}} = \left(\frac{A_{i,j}}{A_{i,k}}\right)^{-\frac{e+1}{e+\phi}}, \tag{A.7}$$

$$\frac{y_{i,j}}{y_{i,k}} = \left(\frac{A_{i,j}}{A_{i,k}}\right)^{\phi\frac{e+1}{e+\phi}}. \tag{A.8}$$

For goods with $i \geq \bar{\imath}$, this implies

$$p_{i,N} = \frac{p^{*} i^{\beta_c}}{\alpha} \left[1 + \beta_a \left(\phi - 1\right) \frac{e+1}{e+\phi}\right]^{\frac{1}{\phi-1}}, \tag{A.9}$$

$$p_{i,T} = \frac{p^*}{\alpha} i^{\beta_c}, \tag{A.10}$$

$$p_i = \frac{p^*}{\alpha} \frac{i^{\beta_c}}{\left[1 - w' n_i\right]^{\frac{1}{\phi-1}}}, \tag{A.11}$$

where

$$w' = \frac{\beta_a (\phi - 1) \frac{e+1}{e+\phi}}{1 + \beta_a (\phi - 1) \frac{e+1}{e+\phi}} < 1. \tag{A.12}$$

Proceeding as in the case with endowment, we get

$$c_{i,N} = n_i \left[\frac{1}{n_i} \int_0^{n_i} c_{i,j}^{1-\frac{1}{\phi}} dj\right]^{\frac{1}{1-\frac{1}{\phi}}} = \frac{y_{i,n_i} n_i}{\left[1 + \beta_a (\phi - 1) \frac{e+1}{e+\phi}\right]^{\frac{1}{1-\frac{1}{\phi}}}}, \tag{A.13}$$

$$c_{i,T} = (1 - n_i) \left[\frac{1}{1 - n_i} \int_{n_i}^1 c_{i,j}^{1-\frac{1}{\phi}} dj\right]^{\frac{1}{1-\frac{1}{\phi}}} = (1 - n_i)\, y_{i,n_i} \tag{A.14}$$

$$c_i = \left[n_i \left(\frac{c_{i,N}}{n_i}\right)^{\frac{\phi-1}{\phi}} + (1 - n_i) \left(\frac{c_{i,T}}{1 - n_i}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} = y_{i,n_i} \left[1 - w' n_i\right]^{\frac{\phi}{\phi-1}}. \tag{A.15}$$

The employment in production of traded and nontraded varieties of various goods is given by

$$l_{i,N} = \left(\frac{aA}{W} \frac{p^* i^{\beta_c}}{\alpha}\right)^{1+e} \frac{n_i^{1+\beta_a(1+e)}}{1 + \beta_a (\phi - 1) \frac{e+1}{e+\phi}}, \tag{A.16}$$

$$l_{i,T} = \left(\frac{aA}{W} \frac{p^* i^{\beta_c}}{\alpha}\right)^{1+e} \frac{1}{1 + \beta_a (e + 1)} \left[1 - n_i^{1+\beta_a(1+e)}\right], \tag{A.17}$$

$$l_i = l_{i,N} + l_{i,T}. \tag{A.18}$$

Note that for $i \leq \bar{\imath}$, $l_{i,T} = 0$ as $n_i = 1$. The labor market clearing condition is

$$\int_0^1 l_i di = L \equiv 1, \tag{A.19}$$

which can be explicitly solved to give

$$W^{1+e} = \int_0^1 \left(aA \frac{p^* i^{\beta_c}}{\alpha}\right)^{1+e} \frac{1}{1 + \beta_a (e + 1)} \left[1 + \frac{1 + e}{\phi - 1} w' n_i^{1+\beta_a(1+e)}\right] di. \tag{A.20}$$

Consumer's optimization over different goods i and m, as before, gives

$$\frac{c_i}{c_m} = \left[\frac{p_i}{p_m}\right]^{-\gamma}, \tag{A.21}$$

from which, for $i \geq \bar{\imath}$, one obtains

$$n_i = i^{-\frac{\gamma+e}{1+e}\frac{\beta_c}{\beta_a}} \left[\frac{1 - w'n_i}{1 - w'n_1}\right]^{\frac{1}{\beta_a(1+e)}\frac{\gamma-\phi}{\phi-1}} n_1. \tag{A.22}$$

which also defines $\bar{\imath}$, when n_i is set to 1. The price of the marginal fully nontraded good is

$$p_{\bar{\imath}} = p_{\bar{\imath},N} = \frac{p^* \bar{\imath}^{\beta_c}}{\alpha} \left[1 + \beta_a (\phi - 1) \frac{e+1}{e+\phi}\right]^{\frac{1}{\phi-1}}, \tag{A.23}$$

and, for any other fully nontraded good $i < \bar{\imath}$, consumer optimization, as before, implies

$$p_i = \left[\frac{c_i}{c_{\bar{\imath}}}\right]^{-\frac{1}{\gamma}} p_{\bar{\imath}} = p_{\bar{\imath}}, \quad i \leq \bar{\imath}. \tag{A.24}$$

Solving for equilibrium again involves searching over one unknown. Given n_1, (A.22) can be solved for n_i as function of i, and for $\bar{\imath}$. Given n_i, (A.20) gives the corresponding value of wage rate W. Finally, the value of n_1, the share of non-traded varieties for good 1, is determined by the external balance condition or the budget constraint.

For example, in the steady state case with current account in balance, we again have

$$p_H c_H \equiv \int_0^1 p_i c_i di = \theta \int_0^1 p_i y_i di \equiv \theta p_H y_H, \tag{A.25}$$

where

$$p_i y_i = \left(\frac{aA}{W}\right)^e \left(\frac{p^* i^{\beta_c}}{\alpha}\right)^{1+e} \frac{1}{1 + \beta_a (e+1)} \left[1 + \frac{1+e}{\phi - 1} w' n_i^{1+\beta_a(1+e)}\right], \tag{A.26}$$

and

$$p_i c_i = \left(\frac{aA}{W}\right)^e \left(\frac{p^* i^{\beta_c}}{\alpha}\right)^{1+e} [1 - w n_i]. \tag{A.27}$$

Substituting for $p_i c_i$ and $p_i y_i$ from (A.26-A.27) into (A.25) and noting (1) that for $i \geq \bar{\imath}$ (A.22) gives n_i as function of i and n_1 and $\bar{\imath}$ as a function of n_1, and (2) that (A.20) gives W as function of n_i (and hence n_1), one obtains one equation in one unknown, n_1. In fact, for this steady state case, W drop out of the budget constraint. However, this will not be the case out of the steady state.

Appendix B. A Two Country Model

The two countries, Home and Foreign, are identical in all respects except that their endowment of different varieties of a good is anti-symmetric as mentioned in the paper

$$y_{i,j} = y\left(\frac{1+j}{\delta}\right)^{\beta_a}, \quad y>0, \ \ \beta_a>0, \ \ \delta>0, \tag{B.1}$$

$$y^*_{i,j} = y\left(\frac{2-j}{\delta}\right)^{\beta_a}. \tag{B.2}$$

As usual, variables with asterisk denote quantities for the Foreign. Transport costs are as in the small economy case. The preferences for Home's representative consumer are

$$c^{1-\frac{1}{\gamma}} = \int_0^1 c_i^{1-\frac{1}{\gamma}} di, \tag{B.3}$$

where $\gamma > 1$ is the elasticity of substitution across goods produced by different industries. For every sector there exists a variety n_i such that Home imports varieties $j \in [0, n_i)$, exports varieties $j \in (1-n_i, 1]$, and varieties $[n_i, 1-n_i]$ are nontraded. Thus, c_i, the index of consumption of good i can be broken up as follows

$$\begin{aligned} c_i^{1-\frac{1}{\phi}} &= \int_0^{n_i} c_{i,j}^{1-\frac{1}{\phi}} dj + \int_{n_i}^{1-n_i} c_{i,j}^{1-\frac{1}{\phi}} dj + \int_{1-n_i}^{1} c_{i,j}^{1-\frac{1}{\phi}} dj, \\ &= n_i\left(\frac{c_{i,M}}{n_i}\right)^{1-\frac{1}{\phi}} + (1-2n_i)\left(\frac{c_{i,N}}{1-2n_i}\right)^{1-\frac{1}{\phi}} + n_i\left(\frac{c_{i,X}}{n_i}\right)^{1-\frac{1}{\phi}}, \end{aligned} \tag{B.4}$$

where $\phi > 1$ is the elasticity of substitution among varieties of a good and

$$c_{i,M} = n_i\left[\frac{1}{n_i}\int_0^{n_i} c_{i,j}^{1-\frac{1}{\phi}} dj\right]^{\frac{1}{1-\frac{1}{\phi}}}, \tag{B.5}$$

$$c_{i,N} = (1-2n_i)\left[\frac{1}{1-2n_i}\int_{n_i}^{1-n_i} c_{i,j}^{1-\frac{1}{\phi}} dj\right]^{\frac{1}{1-\frac{1}{\phi}}}, \tag{B.6}$$

$$c_{i,X} = n_i\left[\frac{1}{n_i}\int_{1-n_i}^{1} c_{i,j}^{1-\frac{1}{\phi}} dj\right]^{\frac{1}{1-\frac{1}{\phi}}}. \tag{B.7}$$

Once again, $c_{i,M}$, $c_{i,N}$ and $c_{i,X}$ have the interpretation of 'aggregate' consumption of imported, nontraded and exported varieties of good i and, therefore, $c_{i,M}/n_i$, $c_{i,N}/(1-2n_i)$ and $c_{i,X}/n_i$ have the interpretation of 'average' consumption.

The consumption-based-price indices for the above defined consumption aggregates follow immediately

$$
\begin{aligned}
p^{1-\gamma} &= \int_0^1 p_i^{1-\gamma} di, && \text{(B.8)}\\
p_i^{1-\phi} &= \int_0^{n_i} p_{i,j}^{1-\phi} dj + \int_{n_i}^1 p_{i,j}^{1-\phi} dj, \\
&= n_i p_{i,M}^{1-\phi} + (1-2n_i) p_{i,N}^{1-\phi} + n_i p_{i,X}^{1-\phi}. && \text{(B.9)}
\end{aligned}
$$

Here p is the aggregate price level in Home, p_i is the price index of good i. The price indices of imported, nontraded and traded varieties of good i, $p_{i,M}$, $p_{i,N}$ and $p_{i,X}$ are

$$
\begin{aligned}
p_{i,M} &= \left[\frac{1}{n_i}\int_0^{n_i} p_{i,j}^{1-\phi} dj\right]^{\frac{1}{1-\phi}}, && \text{(B.10)}\\
p_{i,N} &= \left[\frac{1}{1-2n_i}\int_{n_i}^{1-n_i} p_{i,j}^{1-\phi} dj\right]^{\frac{1}{1-\phi}}, && \text{(B.11)}\\
p_{i,X} &= \left[\frac{1}{n_i}\int_{1-n_i}^{1} p_{i,j}^{1-\phi} dj\right]^{\frac{1}{1-\phi}}, && \text{(B.12)}
\end{aligned}
$$

and

$$
p_{i,T}^{1-\phi} = n_i p_{i,M}^{1-\phi} + n_i p_{i,X}^{1-\phi}. \tag{B.13}
$$

Using conditions from consumer optimization, as in the small open economy case, one can derive the prices of all varieties of good i in terms of the price p_{i,n_i} of the *marginal imported variety*, n_i,

$$
\begin{aligned}
p_{i,j} &= \left[\frac{(1+\tau_i)\, y_{i,j} + y_{i,j}^*}{(1+\tau_i)\, y_{i,ni} + y_{i,n_i}^*}\right]^{-\frac{1}{\phi}} p_{i,n_i}, && j \in [0, n_i), && \text{(B.14)}\\
p_{i,j} &= \left[\frac{1+j}{1+k}\right]^{-\frac{\beta_a}{\phi}} p_{i,n_i}, && j \in (n_i, 1-n_i], && \text{(B.15)}\\
p_{i,j} &= \left[\frac{y_{i,j} + (1+\tau_i)\, y_{i,j}^*}{(1+\tau_i)\, y_{i,ni} + y_{i,n_i}^*}\right]^{-\frac{1}{\phi}} \left[\frac{2-n_i}{1+n_i}\right]^{-\frac{\beta_a}{\phi}} p_{i,n_i}, && j \in (1-n_i, 1]. && \text{(B.16)}
\end{aligned}
$$

Recall, due to assumed symmetry *marginal exported variety* is $1-n_i$.

Similarly, consumption of varieties of good i is given by

$$
\begin{aligned}
c_{i,j} &= s_{1,i}\left[\left(1+\tau_i\right)y_{i,j}+y^*_{i,j}\right], && j\in[0,n_i), && \text{(B.17)}\\
c_{i,j} &= y_{i,j}=y\left[\frac{1+j}{\delta}\right]^{\beta_a}, && j\in[n_i,1-n_i], && \text{(B.18)}\\
c_{i,j} &= s_{2,i}\left[y_{i,j}+\left(1+\tau_i\right)y^*_{i,j}\right], && j\in(1-n_i,1], && \text{(B.19)}
\end{aligned}
$$

where

$$
\begin{aligned}
s_{1,i} &\equiv \frac{c_{i,j}}{\left(1+\tau_i\right)y_{i,j}+y^*_{i,j}}=\frac{\left(1+n_i\right)^{\beta_a}}{\left(1+\tau_i\right)\left(1+n_i\right)^{\beta_a}+\left(2-n_i\right)^{\beta_a}}, && j\in[0,n_i), && \text{(B.20)}\\
s_{2,i} &\equiv \frac{c_{i,j}}{y_{i,j}+\left(1+\tau_i\right)y^*_{i,j}}=\frac{\left(2-n_i\right)^{\beta_a}}{\left(2-n_i\right)^{\beta_a}+\left(1+\tau_i\right)\left(1+n_i\right)^{\beta_a}}, && j\in(1-n_i,1]. && \text{(B.21)}
\end{aligned}
$$

From (B.14-B.21), it is easy to see that aggregate consumption of good i and its price are functions of n_i and p_{i,n_i} alone. Explicitly denoting this dependence, consumer optimization across goods with some varieties traded implies

$$\frac{c_i\left(n_i\right)}{c_1\left(n_1\right)}=\left[\frac{p_i\left(n_i,p_{i,n_i}\right)}{p_1\left(n_1,p_{1,n_1}\right)}\right]^{-\gamma}. \tag{B.22}$$

A similar condition holds for Foreign[14]

$$\frac{c^*_i\left(n_i\right)}{c^*_1\left(n_1\right)}=\left[\frac{p^*_i\left(n_i,p^*_{i,n_i}\right)}{p^*_1\left(n_1,p^*_{1,n_1}\right)}\right]^{-\gamma}. \tag{B.23}$$

As in case of small open economy, there is a *marginal fully nontraded good*, $\bar{\imath}$, with $n_{\bar{\imath}}=0$, which solves

$$\frac{c_{\bar{\imath}}\left(1\right)}{c_1\left(n_1\right)}=\left[\frac{p_i\left(0,p_{\bar{\imath},0}\right)}{p_1\left(n_1,p_{1,n_1}\right)}\right]^{-\gamma}, \tag{B.24}$$

and for goods with $i<\bar{\imath}$,

$$p_{i,j}=p_{\bar{\imath},j}\quad j\in[0,1]. \tag{B.25}$$

To begin solving for the equilibrium, first normalize $p_{1,n_1}=1$. Also note that for the marginal imported variety for Home, n_i,

$$p_{i,n_i}=\left(1+\tau_i\right)p^*_{i,n_i}. \tag{B.26}$$

In light of (B.26), given n_1, (B.22-B.23) can be solved for n_i and p_{i,n_i} for every $i>\bar{\imath}$

[14]One can derive conditions similar to (B.14-B.21) for Foreign which will result in functional dependence of c^*_i and p^*_i on n_i assumed in (B.23).

where value of $\bar{\imath}$ follows from (B.24). Also, recall for $i \leq \bar{\imath}$, $n_i = 0$. The appropriate value of n_1 is found by imposing the budget constraint which on simplification gives[15]

$$\int_{\bar{\imath}(n_1)}^{1} p_i\left(n_i, p_{i,n_i}\right) c_i\left(n_i\right) di = \int_{\bar{\imath}(n_1)}^{1} p_i\left(n_i, p_{i,n_i}\right) y_i di. \tag{B.27}$$

Note that, again, to solve for the entire equilibrium, one needs to solve only one equation (B.27) in one unknown n_1. However, if the two countries were dissimilar besides having anti-symmetric endowments, one will need to solve two equations in two unknowns, the marginal imported and exported variety of good 1 for Home, n_1^m and n_1^x. In the symmetric case discussed here, $n_1 = n_1^m = 1 - n_1^x$.

[15]It is different from external balance condition as some varieties are nontraded for $i > \bar{\imath}$.

References

Baldwin, R. and Forslid, R. 2010. Trade Liberalization with Heterogeneous Firms. Review of Development Economics 14(2), 161-176.

Bergin, P. and Glick, R., 2007a. Endogenous Nontradability and its Macroeconomic Implications for Current Account. Review of International Economics 15(5), 916-931.

Bergin, P. and Glick, R. 2007b. Tradability, Productivity, and International Economic Integration. Journal of International Economics 73, 128-151.

Bergin, P. and Glick, R. 2009. Endogenous Tradability and its Macroeconomic Implications. Journal of Monetary Economics 56, 86-95.

Bernard, A., Redding, S., and Schott, P. 2007. Comparative Advantage and Heterogeneous Firms. Review of Economic Studies 74, 31-66.

Chaney, T. 2008. Distorted Gravity: The Intensive and Extensive Margins of International Trade. American Economic Review 98(4), 1707–1721.

Crucini, M., Telmer, C., and Zachariadis, M. 2005. Understanding European Real Exchange Rates. American Economic Review 95, 724-738.

Dornbusch, R., Fisher, S., and Samuelson, P. 1977. Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods. American Economic Review 67, 823-839.

Eaton, J. and Kortum, S. 2002. Technology, Geography, and Trade. Econometrica 70, 41-69.

Ghironi, F. and Melitz, M. 2005. International Trade and Macroeconomic Dynamics with Heterogeneous Firms. Quarterly Journal of Economics, 865-915.

Hummels, D. 1999. Toward a Geography of Trade Costs. Working paper, Purdue University.

Hummels, D. 2001. Time as a Trade Barrier. Working paper, Purdue University.

Johnson, R. 2014. Trade in Intermediate Inputs and Business Cycle Comovement. American Economic Journal: Macroeconomics 6(4), 39–83.

Melitz, M. 2003. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. Econometrica 71, 1695–1725.